

# The nucleon and the two solar mass neutron star

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The existence of a star with such a large mass means that the equation of state is stiff enough to provide a high enough pressure up to a fairly large central densities,. Such a stiff equation of state is possible if the ground state has nucleons as its constituents. This further implies that a purely nucleon ground state may exist till about four times nuclear density which indicates that quarks in the nucleon are strongly bound and that the nucleon nucleon potential is strongly repulsive. We find this to be so in a chiral soliton model for the nucleon which has bound state quarks. We point out that this has important implications for the strong interaction  $\mu_B$  vs T phase diagram.

## INTRODUCTION

Equations of state (EOS) for stable stars like white dwarfs that involve nonrelativistic electrons counteract gravitational infall of matter through a fermi pressure that is proportional to the density to the (5/3) power. When the mass of the star increases so does the electron density making the electron fermi energy relativistic. Fermi pressures of relativistic electrons are proportional to density to the (4/3) power and cannot hold up to gravitational pressure. This is the Chandrasekhar instability that sets the maximum mass of such stars.

However, for neutron stars, even a pure nonrelativistic fermi gas of neutrons is not sufficient to give large masses . Such a non interacting nonrelativistic fermi gas can give stable neutron stars of maximum mass about 0.7 solar mass - this a general relativistic effect coming from the Oppenheimer – Volkoff equation. Beyond this mass the pressure needs to be more repulsive than just the fermi pressure . This enhanced pressure is provided by nuclear interactions like the hard core.

It is known that there are many purely nonrelativistic nucleon based neutron star models that have neutron stars with maximum mass above 2 solar masses, eg. the APR 98 EOS of Akmal, Pandharipande and Ravenhall [1]. For a brief review of nuclear stars and their EOS we refer the reader to [2, 3]. It is also known that conventional hybrid stars with soft, relativistic quark matter cores surrounded by a nonrelativistic n+p+e plasma in beta equilibrium generally give a maximum mass for neutron stars of only  $\sim 1.6$  solar mass [2, 4].

The recent [5] discovery of  $\simeq 2$  solar mass (highest mass) neutron star, the recycled binary pulsar PSR J1614-2230, using the precision technique of Shapiro delay confronts us with question of what is the constituent profile of such a star. This issue was first raised in [6]

In view of the foregoing, we investigate the following question; if matter in neutron stars is entirely composed

of non relativistic nucleon degrees of freedom then can we have a simple resolution of this question?

It is useful to recall that the recycled binary pulsar, PSR J 1614-2230, is rotating fast at a period of 3 millisecond and we expect a  $\sim 10\%$  diminution of the central density from the rotation from centrifugal forces [7]. Since APR 98 [1] reports results for static stars, we expect the central density of a fast rotating 1.97 solar mass star to be approximately  $\sim$  the central density of a static 1.8 solar mass star.

## THE MAXWELL CONSTRUCTION BETWEEN NUCLEAR MATTER AND QUARK MATTER

We work in an effective chiral symmetric theory that is QCD coupled to a chiral sigma model. The theory thus preserves the symmetries of QCD. In this effective theory chiral symmetry is spontaneously broken and the degrees of freedom are constituent quarks which couple to colour singlet, sigma and pion fields as well as gluons. Furthermore, since we do not have exact solutions for a theory of the strong interactions, we work in Mean field theory. The nucleon in such a theory is a colour singlet quark soliton with three valence quark bound states [11]. The quark meson couplings are set by matching mass of the nucleon to its experimental value and the meson self coupling is set from pi-pi scattering, which in turn sets the tree level sigma particle mass to be of order 800 MeV. Such an effective theory has a range of validity up to centre of mass energies ( or quark chemical potentials) of  $\sim 800$  MeV. For details we refer the reader to ref. [4].

This is one of the simplest effective chiral symmetric theory for the strong interactions at intermediate scale and we use this consistently to describe, both, the composite nucleon of quark bound states and quark matter. We expect it to be valid till the intermediate scales quoted above. Of course inclusion of the higher mesonic

degrees of freedom like the  $\rho$  and A1 would make for a more complete description. We work at the mean field level where the gluon interactions are subsumed in the colour singlet sigma and pion fields they generate. We could further add perturbative gluon mediated corrections but they do not make an appreciable difference.

One of lowest energy ground states at high baryon density that we find in such chiral models is a neutral pion condensed state [8, 9]. The equation of state for neutron stars for such a state has been obtained in [4, 10] A simple way to look at whether nucleons can dissolve into quark matter is to plot  $E_B$ , the energy per baryon in the ground state of both, the quark matter and the nuclear phases, versus  $1/n_B$ , where  $n_B$  is the baryon density. For the quark matter equation of state see Fig.1 [4] in which the quark matter EOS is indicated by the solid curves and the APR [1] non relativistic nucleon EOS by the dashed line. The slope of the common tangent between the two phases then gives the pressure at the phase transition and the intercept, the common baryon chemical potential.

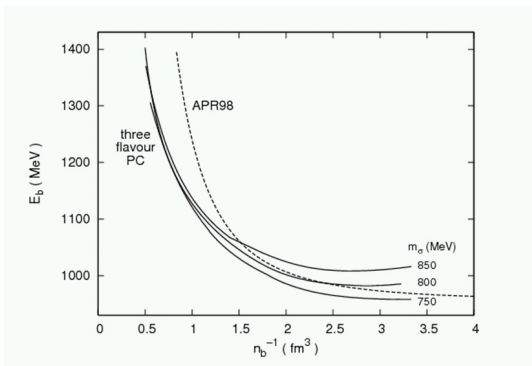


FIG. 1. The Maxwell construction: Energy per baryon plotted against the reciprocal of the baryon number density for APR98 equation of state (dashed line) and the 3-flavour pion-condensed phase (PC) for three different values of  $m_\sigma$  (solid lines). A common tangent between the PC phase and the APR98 phase in this diagram gives the phase transition between them. The slope of a tangent gives the negative of the pressure at that point, and its intercept gives the chemical potential. As this figure indicates, the transition pressure moves up with increasing  $m_\sigma$ , and at  $m_\sigma$  below  $\sim 750$  MeV a common tangent between these two phases cannot be obtained. (From Fig. 2 of Soni and Bhattacharya [4] )

As can be seen from Fig.1, it is the tree level value of the sigma mass that determines the intersection of the two phases; the higher the mass the higher the density at which the transition to quark matter will take place. In [4] it was found that above,  $m_\sigma \sim 850$  MeV, stars with quark matter cores become unstable as their mass goes up beyond the allowed maximum mass. So, if we want purely nuclear stars we should, in this model, work at,  $m_\sigma \geq 850$  MeV [4].

From Fig. 1, for the tree level value of the sigma mass  $\sim 850$  MeV, the common tangent in the two phases

starts at  $1/n_B \sim 1.75 \text{ fm}^3$  ( $n_B \sim 0.57/\text{fm}^3$ ) in the nuclear phase of APR [A18 + dv +UIX] [1] and ends up at  $1/n_B \sim 1.25 \text{ fm}^3$  ( $n_B \sim 0.8/\text{fm}^3$ ) in the quark matter phase.

At the above densities between the two phases there is a mixed phase at the pressure given by the slope of the common tangent and the at a baryon chemical potential given by the intercept of the common tangent on the vertical axis. If we are to stay in the nuclear phase the best way is to look at the central density of the nuclear (APR) stars and if it so happens that the central density is lower than that at which the above phase transition begins then we can safely say that the star remains in the nuclear phase.

Going back to the APR phase in in fig 11 of APR [1] we find that for the APR [A18 + dv +UIX] the central density of a star of 1.8 solar mass is  $n_B \sim 0.62/\text{fm}^3$ , very close to the initial density at which the phase transition begins.

The reason we are taking a static star mass of 1.8 solar mass from APR [1] is that for PSR-1614, the star is rotating fast at a period of 3 millisecond and we expect a  $\sim 10\%$  diminution of the central density from the rotation [7]. Equivalently, since the above paper reports results for static stars, the central density of a fast rotating 1.97 solar mass star  $\sim$  the central density of a static 1.8 solar mass star.

Now we have found that in above scenario the central density is of the same order as the density at which the above phase transition begins in the nuclear phase. Ideally we would like the central density to be a little less than the initial density at which the above phase transition begins in the nuclear phase.

## BEYOND THE MAXWELL TANGENT CONSTRUCTION FOR THE PHASE TRANSITION

How do we change the crossover and Maxwell tangent construction for the phase transition? There are two ways of moving the crossover between the 2 phases (and also the initial density at which the above phase transition begins ) in the nuclear phase to higher density.

(i) By increasing the tree level mass of the sigma we can move the quark matter curve up (Fig. 1), thus moving the initial density at which the above phase transition begins in the nuclear phase to higher density. However we have to be careful. There is not much freedom here, as this is what also determines the  $\pi - \pi$  scattering.

(ii) At 3 - 4 times nuclear density  $n_B \sim 0.6/\text{fm}^3$  the average energy per baryon in the APR EOS is less than 1050 MeV. However, the fermi energy may be close to or above the lambda particle mass, which will make it possible to have a small admixture of hyperons.

This will soften the nuclear EOS at high density. The

phase transition then begins in the nuclear phase at higher density, but this will also reduce the maximum mass. We have to ensure that the the maximum mass stays above two solar masses. There is an extensive literature on the effect of hyperons at well above ( 3 -4 times) nuclear density but with no definite conclusions. We thus do not pursue this question further here.

However, the Maxwell construction is not the final word on the phase transition. In any case the above analysis assumes point particle nucleons. It does not take account of the structure and the quark binding inside the nucleon ( which depends mainly on the quark meson coupling ) or the nucleon nucleon repulsion as we squeeze them. This is not captured by the Maxwell construction. We now go on to show that this could move the transition from the nuclear to the quark phase to appreciably higher density.

### Binding energy of a quark in a nucleon

An appoximate and simple expession for the energy of a colour singlet nucleon soliton with three coloured bound state quarks is given below, [11]. Here ,  $g$  is quark meson (Yukawa) coupling,  $f_\pi$ , the pion decay constant and,  $N$ , the number of bound state quarks. We shall work with the dimensionless parameter,  $X = Rgf_\pi$ , where  $R$  is the soliton radius. This follows from a simple parametrization for a soluble model (see fig. 2). The 'mass' of a 'free' quark in this model is given by,  $m_q = gf_\pi$ ,

$$E/(gf_\pi) = \left( \frac{3.12}{X} N - 0.94.N \right) + 24 \frac{X}{g^2} \quad (1)$$

Minimizing this with respect to ,  $X$

$$X^2 = \frac{3.12g^2N}{24} \quad (2)$$

On substitution of this value

$$E_{min}/(gf_\pi) = \left( \sqrt{\frac{3.12N.24}{g^2}} \right) - 0.94N \quad (3)$$

For the nucleon we must set ,  $N = 3$  as all three quarks sit in the bound state. We can now evaluate the coupling,  $g$ , by setting the nucleon mass to  $960MeV$ . This yields a value for ,  $g \sim 6.9$ .

However, the above formula allows us to look at the energy of the configuration in which two quarks sit in the bound state and one is moved up to the continuum. Such a state will give a measure of the energy required to unbind the nucleon.

We note that in this mean field model there is no confinement: in any case, close to the quark matter phase transition density, the nucleons will go into quark matter

at the transition and not 'free' quarks making confinement a peripheral issue.

We can easily check the possible bound states by evaluating the ratio of the energy of bound states with 2 and 3 quarks, which is given by,  $E_{min}/(gf_\pi)$  and the respective number of unbound ( 'free' )quarks. The 'mass' of a 'free' quark in this model is given by,  $m_q = gf_\pi$ , This is simply done by dividing the above equations by,  $N$ : if the answer is less than ,1, we have a bound state, otherwise not.

$$\begin{aligned} E_{min}/(Ngf_\pi) &= \left( \sqrt{\frac{3.12N.24}{g^2}} \right) - 0.94N \\ &\sim 0.5 \text{ for } N = 3 \\ &\sim 0.83 \text{ for } N = 2 \\ &\sim 1.27 \text{ for } N = 1 \end{aligned} \quad (4)$$

indicating that regardless of the value of  $f_\pi$  we have bound states for  $N = 2$  and  $3$ . Given the value of ,  $g \sim 6.9$ , we can find the energy required to unbind a quark from such a nucleon. The energy of a two quark bound state and an unbound quark is  $1707 MeV$  in comparision to the energy of a 3 quark bound state nucleon which is ,  $960 MeV$ .

i) The difference gives the binding energy of the quark in the nucleon,  $745 MeV$ . The quark binding in this model is very high. In this model the quark bound state eigenvalue (Fig. 2) [11] is well described by the figure given below.

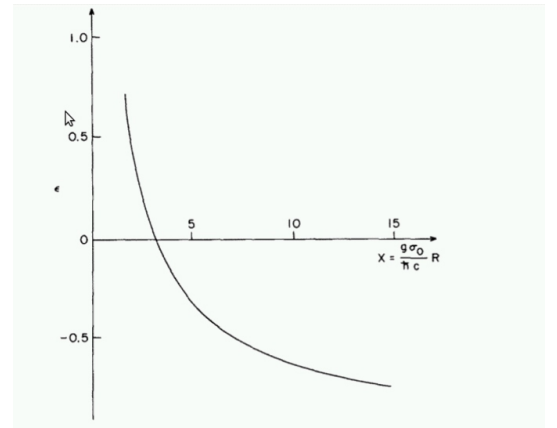


FIG. 2. Dependence of the quark energy on the soliton size  $X$  in the quark soliton model  
(From Fig. 2 of Kahana, Ripka and Soni [11])

ii) We can see that the quarks will become unbound ( go to the continuum) when the energy eigenvalue is larger than the unbound mass of the quark which is given by  $m_q = gf_\pi$ . This happens when in the dimensionless units used in Fig. 2

$$\epsilon \geq 1, \text{ at } X = 3.12/1.94 = 1.6. \quad (5)$$

This translates into  $R = (1.6/2.5) \text{ fm}^{-1} \sim 0.6 \text{ fm}^{-1}$ .

This is the effective radius of the squeezed nucleon at which the bound state quarks are liberated to the continuum. By inverting the volume occupied by the nucleon and assuming hexagonal close packing, this translates to nucleon density of

$$1/(6R^3) \sim 0.77 \text{ fm}^{-3} \quad (6)$$

Thus the quark bound states in nucleon persist until a much higher density  $\sim 0.8/\text{fm}^3$ . In other words, nucleons can survive well above the density at which the Maxwell phase transition begins and appreciably above the central density of the APR 2-solar-mass star.

### Nucleon Nucleon repulsion

Another feature is the the nucleon nucleon potential. It has been found for skyrmions and such quark solitons with skyrmion configurations that there is a strong N-N repulsion that forces the lowest baryon number  $N_B = 2$  configuration to become toroidal [12]. This is an indication that nucleon nucleon potential becomes strongly repulsive.

It thus follows that the phase transition from nuclear to quark matter will encounter a potential barrier before the quarks can go free. This effect cannot be seen by the coarse Maxwell construction which does not track their transition.

## DISCUSSION AND CONSEQUENCES

These considerations will modify the simple minded Maxwell construction above. It follows from above that the internal structure of the nucleon will move the phase transition to higher density. All in all this produces a very plausible scenario of how the  $\sim 2$  solar mass star can be achieved in a purely nuclear phase.

A possible consequence of this unexpected scenario at high density is that the the phase diagram of QCD which plots temperature on the y axis versus baryon chemical potential on the x axis, the quark matter transition for finite density ( in the range above) will be lifted up along the temperature axis.

For example, if for zero baryon density we have chiral restoration at  $T_\Xi \sim 150 \text{ Mev}$ , then at small baryon density such a temperature will probably not be able to dissociate the nucleon bound state that lives in a chirally broken (SBCS) ground state, as its binding energy is very large.

Now we present a heuristic way of determining the energy cost of maintaining a nucleon as a quark bound state

soliton (in an island of spontaneously broken chiral symmetry) at this temperature. First, we estimate the thermal energy in a volume of a nucleon of radius,  $R \sim 1 \text{ fermi}$ , which is approximately,  $\sim \text{Volume} \cdot (kT_\Xi)^4 \sim 250 \text{ Mev}$ . We must also add the cost in gradient energy of decreasing the meson VEV's from  $f_\pi$  inside of a soliton nucleon to 0 (the chiral symmetry restored value), outside of the nucleon. If we assume this happens over a typical length scale of,  $\sim 1 \text{ fermi}$ , the gradient energy also works out to be,  $\sim 200 - 250 \text{ Mev}$ . The sum of these energies is around  $400 - 500 \text{ Mev}$ , whereas, the binding energy of the quark in such a nucleon is  $\sim 750 \text{ Mev}$ , indicating that at chiral restoration,  $T_\Xi \sim 150 \text{ Mev}$ , the nucleon may yet be intact.

Thus, at finite but small baryon density and  $T_\Xi \sim 150 \text{ Mev}$ , there may emerge a new intermediate mixed phase in which nucleons will exist as bound states of locally spontaneously broken chiral symmetry (SBCS) in a sea of chirally restored quark matter. Such a low baryon density state could be seen in lattice calculations. This is quite the opposite to the popular bag notions of the nucleon as being islands of restored chiral symmetry in a SBCS sea.

We also note that in the chirally restored state, the quarks acquire a typical temperature dependent pole mass proportional to,  $gT$  in perturbation theory where  $g$  is the QCD coupling constant. The QCD coupling is still strong,  $\frac{g^2}{4\pi} \geq 1$ . For example, if at zero baryon density we have chiral restoration at,  $T_\Xi \sim 150 \text{ Mev}$ , the 3 quarks that make up a baryon have an,  $E_B \geq 1600 \text{ Mev}$ . This will also influence the transition to quark matter.

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- [1] A. Akmal, V. R. Pandharipande and D. G. Ravenhall, Phys. Rev. C **58** (1998) 1804.
  - [2] J. M. Lattimer and M. Prakash, Astrophysical J. **550** (2001) 426;
  - [3] V. Soni and N. D. Haridass, MNRAS **425** (2):(2012) 1558-1566.
  - [4] V. Soni and D. Bhattacharya, Phys. Lett. B **643** (2006) 158.
  - [5] Paul Demorest, Tim Pennucci, Scott Ransom, Mallory Roberts, Jason Hessels Nature **467**, (2010)1081-1083
  - [6] V. Soni, P. Haensel and M. Rosina, Bled Workshops in Physics **13**, No 1 (2012) 42, also available at <http://www-fl.ijs.si/BledPub>.
  - [7] Haensel et al., New Astronomy Reviews **51** (2008) 785; arXiv:0805.1820; arXiv:1109.1179 v2.
  - [8] Dautry, F. and Nyman, E. M. 1979, Nucl. Phys. A **319** 323
  - [9] Kutschera, M., Broniowski, W. and Kotlorz, A. 1990, Nucl. Phys. A **516**, 566
  - [10] V. Soni and D. Bhattacharya, arXiv. hep-ph/0504041 v2.
  - [11] S. Kahana, G. Ripka and V. Soni, Nuclear Physics A **415** (1984) 351.

- [12] M. S. Sriram et al., Phys. Lett. B **342** (1995) 201.